The convergence of the numerical solution of system (2.2) to an exact value degrades as the ratio of side crack length to main crack length decreases. Therefore in the limiting case as $l_1/l \to 0$ a numerical solution cannot be obtained directly. For a two-branch broken crack (Fig. 6, solid lines) and a three-branched crack (Fig. 6, dashed lines) the values of $K_{ij}(\alpha)$ were calculated by extrapolation from numerical data obtained for $l_1/l = 0.01, 0.02$.

Similar functions $K_{ij}(\alpha)$ were presented in [11]. In the case of the broken crack there is quite good agreement between the results obtained and the data of [11] (maximum relative deviation does not exceed 6%), with significantly greater differences for the branching crack. For this last case, [12] presents the dependence of stress intensity coefficients on angle α for $l_1/l = 0.1$. We note that for such an l_1/l value the intensity coefficients calculated by solution of Eq. (2.2) practically coincide with the data of [12].

LITERATURE CITED

1. A. M. Lin'kov, "Integral equations of elasticity theory for a plane with sections loaded by an equalizing system of forces," Dokl. Akad. Nauk SSSR, 218, No. 6 (1974).
2. M. P. Savruk, "Curvilinear crack system in an elastic body with various boundary conditions on the crack edges," Fiz. Khim. Mekh. Mater., 14, No. 6 (1978).
3. P. S. Theocaris and N. I. Iokimidis, "The symmetrically branched crack in an infinite elastic medium," Z. Angew. Math. Phys., 27, No. 6 (1976).
4. V. V. Panasyuk, M. P. Savruk, and A. P. Datsishin, Stress Distribution around Cracks in Plates and Shells [in Russian], Naukova Dumka, Kiev (1976).
5. N. I. Muskhelishvili, Singular Integral Equations [in Russian], Fizmatgiz, Moscow (1962).
6. A. I. Kalandia, "Notes on the singularity of elastic solutions near angles," Prikl. Mat. Mekh., 33, No. 1 (1969).
7. B. M. Zinov'ev and T. F. Karmanova, "Consideration of singularities in numerical solution of elasticity theory problems," Tr. NIIZhTa, No. 190/3 (1978).
8. F. Erdogan and G. D. Gupta, "On the numerical solution of singular integral equations," Q. Appl. Math., 29, No. 4 (1972).
9. G. P. Cherepanov, Mechanics of Brittle Failure [in Russian], Nauka, Moscow (1974).
10. A. V. Boiko and L. N. Karpenko, "Limiting state of a plate weakened by a crack of complex form in biaxial tension," Prob. Prochn., No. 8 (1980).
11. B. A. Bilby, G. E. Cardew, and I. C. Howard, "Stress intensity factors at the tips of kinked and forked cracks," in: Advances in Research in Strength and Fracture of Materials, Fourth International Conference on Fractures, Waterloo, 1977, Vol. 3a, New York (1978).
12. K. Kitagawa and R. Yuyuki, "Stress intensity coefficients in problems of branching cracks in the plane-stressed state," Nikhon Kikai Gakkai Rombunsyu, 41, No. 346 (1975).

OPENING OF A NATURAL MACROCRACK

A. P. Vladimirov and V. V. Struzhanov

UDC 539.375

The shortcomings of the simplest models of macrocracks have been noted many times in the literature. Attempts to construct complete models reduce to selecting some hypothesis concerning the behavior of the medium at the tips of the crack [1, 2], but the process of formation of real macrocracks was not given the proper attention.

A model of natural macrocracks, which takes into account the presence of residual compressive stresses arising at the tip of a crack as it is formed and opposing the opening up of the macrocrack, was proposed in [3, 4]. The purpose of this investigation is to provide experimental justification of the model proposed.

1. To investigate the mechanisms involved in opening up of a natural macrocracks, we prepared a rectangular specimen consisting of SO-95 Plexiglas, to which we gave a matted

Sverdlovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 147-150, March-April, 1983. Original article submitted January 13, 1982.

0021-8944/83/2402-0271$07.50

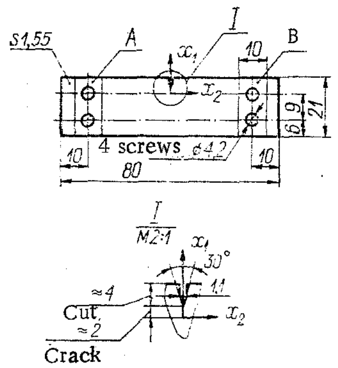

Fig. 1

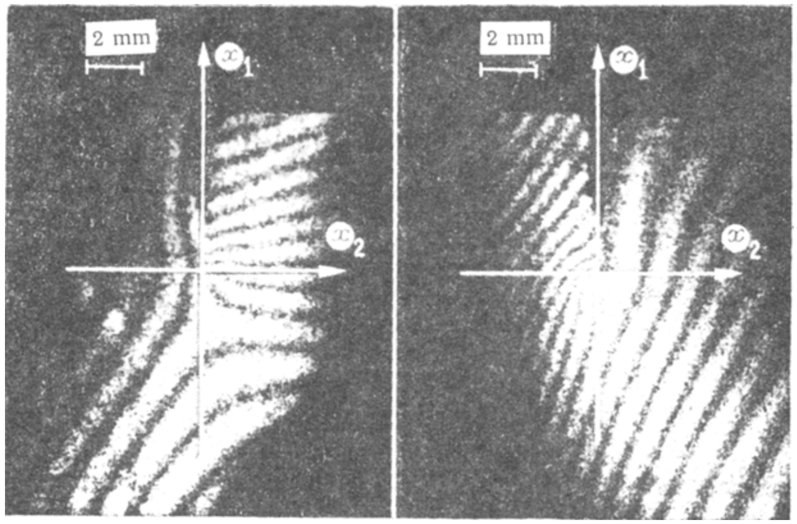

Fig. 2

finish using an abrasive with an average particle size of 20 μm. To increase the coefficient of reflection, we deposited aluminum on the surface of the specimen forming a film with thickness $(5-10)\cdot10^{-6}$ mm. We formed a macrocrack by placing a precut specimen into a loading apparatus and gradually stretched it until a crack appeared; we then removed the load in order to stop the crack. Figure 1 shows the specimen and its dimensions; the origin of coordinates coincides with the tip of the crack. The part A of the specimen was clamped to the stationary clamp of the loading setup and part B was clamped to the movable clamp.

We performed the experiment as follows. First we made a hologram of the specimen with zero load. Then, by moving the movable clamp, we deformed the specimen by an amount $0.39\cdot10^{-3}$ and made a second hologram. The photographic plate was then replaced and the first exposure was made with a deformation of $0.39\cdot10^{-3}$ and the second with a deformation of $0.77\cdot10^{-3}$, etc. In this manner, we obtained from five to ten double-exposure holograms, bringing the deformation of the specimen up to 0.7-0.8 of the deformation at which the crack opened up.

The deformation was performed with a step of $0.39\cdot10^{-3}$ because an optimum spacing of bands for measurements was realized in this case and, in addition, for an initial deformation less than $0.39\cdot10^{-3}$, the interference bands are continuous over the entire surface and therefore the sensitivity of the method in this experiment did not permit observing breakdown of the continuity of the medium under such deformations.

To check the results, the experiment was performed on another specimen. The experiments on both specimens were repeated several times, i.e., several series of holograms were prepared for each specimen.

We created the holograms in countermoving beams [5]. At the image reconstruction stage, we used the technique described in [6, 7], which permits separating and measuring any component of the displacement vector of points on the surface, namely, we photographed the pattern of interference bands through two optical systems, whose axes were situated in the x_2Ox_3 plane and emanate from the origin of coordinates forming angles $\beta_2 = 65°$ and $\beta_2' = 115°$ with the Ox_2 axis, i.e., symmetrical relative to the Ox_3 axis. Typical patterns of bands are shown in Fig. 2. Figure 2a corresponds to an angle of 65° and Fig. 2b corresponds to 115°. The deformation of the specimen is $1.54\cdot10^{-3}$.

We note that we obtained the same results for specimens without the aluminum deposited on them. A bright image is necessary for reliable quantitative investigations.

2. Analysis of the holograms obtained showed that for deformations less than $0.77\cdot10^{-3}$ the crack opens up only partially and then it forms completely. To determine the width of the crack, we used one of the last series of holograms, obtained during the experiment. According to [7], we have

$$\Delta u_2 = \lambda\,(\Delta N_2 - \Delta N_2')/(2\cos\beta_2), \tag{2.1}$$

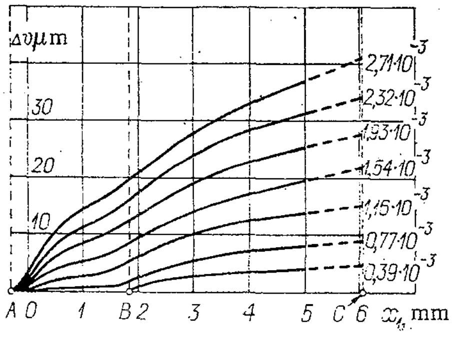

Fig. 3

where Δu_2 is the projection of the displacement vector on the Ox_2 axis; λ is the wavelength of the laser radiation, equal to 0.633 μm; ΔN_2 and $\Delta N_2'$ are determined along the crack from the band patterns obtained with the first and second optical systems, respectively, and in addition the transition from maximum illuminance to the neighboring minimum illuminance in the band pattern corresponds to a change of 0.5 in these quantities.

For each side of the crack, we contructed two combined graphs of the dependences of ΔN_2 and $\Delta N_2'$ on the coordinate x_1. We connected the experimental points, i.e., the coordinates of the maxima and minima in illuminance, by continuous curves. We substituted the difference $\Delta N_2 - \Delta N_2'$ between two curves at a fixed point into Eq. (2.1). The points were chosen with a step 0.1 mm. Subtracting from the displacements obtained the displacement of the crack as a whole, which we took as the displacement of its tip along the Ox_2 axis, we obtained the values Δv_1 and Δv_2 of the width of the crack corresponding to the right and left sides of the crack. The data obtained are presented in Fig. 3, where $\Delta v = (1/2)[\Delta v_1(x_1) + \Delta v_2(x_1)]$, OA is the magnitude of the increment to the growth of the crack, OB is the length of the natural macrocrack, and BC is the length of the cutout. The deformation of the specimen corresponding to each curve is indicated.

3. The incomplete opening of a natural macrocrack with small deformations is explained, evidently, by the fact that as the crack is formed, forces arise that oppose the separation of the sides of the crack. To determine the nature of these forces, we performed the following experiments. Nine specimens with cuts were prepared from the same Plexiglas. We placed each specimen into the loading apparatus and made the first hologram. We then formed a crack, after which we removed the load and made another hologram. We repeated this procedure many times, enlarging the crack each time by 1-2 mm.

In all of the experiments, we obtained interference bands which showed a discontinuity in crossing the crack. In this manner, the residual displacements, which are in fact the reason for the appearance of the experimentally observed bands, are not continuous and for this reason the material studied is not continuous. It follows from here that after the formation of the natural macrocrack and subsequent removal of the load, the atomic interaction forces do not reappear since the sensitivity of the method is such that if an interaction arose between atoms situated on opposite sides of the crack, then it would have been impossible to observe the breakdown in continuity, namely the discontinuity in the displacements.

Further investigation of the band patterns shows that the bands are distributed nonuniformly over the surface of the specimen. They are observed to concentrate near the tip of the crack, in the so-called loosening zone, i.e., the field of residual displacements and, therefore, the field of residual deformations is not uniform.

The appearance of nonuniform residual deformations is caused by the fact that deformations arising in the body during the process of formation of macrocracks, initially, do not satisfy the conditions of compatibility due to the different degree of deformation of the material in the loosening zone and in the undamaged region. The material situated in the loosening zone strives to occupy a large volume, while the undamaged material surrounding it opposes this. As a result, compressive self-balanced stresses appear in the body, which we shall call induced stresses and which oppose the opening up of the crack, appearing as a unique reaction of the material to the fracture.

The presence of induced stresses, opposing the opening of the crack, is likewise confirmed by the fact that the results obtained above could not be reproduced on a specimen with a natural crack, which lay for about a year under room conditions. Under the same conditions as in the experiment, the crack opened up with deformations much smaller than $0.39 \cdot 10^{-3}$. This fact is evidently explained by the relaxation of induced stresses.

We note in conclusion that for small deformations, a natural macrocrack does not open up completely, opening up of the crack is opposed by the compressive induced stresses, and since a loosening zone evidently accompanies the formation of a natural crack in any material, the results of the present work are apparently also valid for a wider class of materials.

LITERATURE CITED

1. V. V. Panasyuk, Limiting Equilibrium of Brittle Solids with Fractures, Mgmt. Info. Serv. (1969).
2. H. Libowitz (ed.), Fracture. Mathematical Foundations of the Theory of Fracture, [Russian translation], Vol. 2, Mir, Moscow (1975).
3. S. D. Volkov and V. P. Stavrov, Statistical Mechanics of Composite Materials [in Russian], Minsk (1978).
4. S. D. Volkov, "Theory of macrocracks. Report No. 2. Models of class M," Probl. Prochn., No. 3 (1981).
5. Yu. I. Ostrovskii, M. M. Butusov, and G. V. Ostrovskaya, Holographic Interferometry [in Russian], Nauka, Moscow (1977).
6. V. M. Ginzburg and B. M. Stepanov (eds.), Holography. Methods and Apparatus [in Russian], Sov. Radio, Moscow (1974).
7. A. P. Vladimirov, "Application of thick photoemulsion layers in holographic interferometry of natural cracks," VINITI, Dep. No. 3577-80 (1981).

EXISTENCE OF SOLUTIONS IN DYNAMICS PROBLEMS OF ONE-DIMENSIONAL PLASTIC STRUCTURES

A. M. Khludnev

UDC 539.214+539.374+517.9

The distinctive feature in the formulations of elastic–plastic and rigid–plastic problems is the presence of an inequality connecting the plastic strain rate and the magnitude of the running stresses. This inequality, called the Mises maximum principle, includes r plastic strain rate components (r depends on the dimensionality of the problem), where it is arranged so that it actually replaces r equations and the system of governing relationships is hence closed. Therefore, it turns out that upon the assignment of initial and boundary conditions, the rates and stresses are determined at each point, and moreover uniquely. Let us note that a corollary of the mentioned inequality that describes the proportionality between the plastic strain component and the components of the flow surface gradient is often used in finding the approximate solutions (by a numerical or analytic method). As a rule, this results in openness of the system of equations. In this sense the utilization of the maximum principle in its initial form is more preferable despite the fact that the inequality itself is a corollary of the more general Drucker postulate. In particular, formulation of the problem by using the inequality was examined in [1], which permitted setting up the solvability of the three-dimensional dynamic elastic–plastic problem.

Generalized stresses (forces, moments, etc.) and strain rates of the middle surface take part in the formulation of elastic–plastic and rigid–plastic problems for thin-walled structures of the shell, plate, and beam type. They are also interrelated by using inequalities [2, 3]. Definite progress has been achieved in the investigation of problems of this kind from the viewpoint of an approximate description of the strain processes. This concerns the case of one space variable especially (see the survey [4]). However, despite the large number of papers on this topic, in practice there are no results referring to the investigation of the correctness in the formulations of such problems. Boundary-value problems for one-dimensional elastic–plastic and rigid–plastic structures are considered in this paper, and results on solvability are formulated.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 150-156, March-April, 1983. Original article submitted April 28, 1982.

0021-8944/83/2402-0274$07.50